Planetary Alignment and Kepler's Law of Periods

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Topics

Physics, Mathematics, Modeling,

Planetary Alignment, Kepler's Law of Periods

Ages

Grades 7-12

Duration

Planetary Alignment: 30 minutes;

Kepler's Law of Periods: 30 minutes

Classroom Application:

Planetary "Alignment" and Kepler's Law of Periods

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Introduction

The sun, moon, planets and stars have fascinated mankind for millennia. Horoscopes and astrology have attempted to correlate these heavenly bodies with human affairs. Of special astrological interest is the alignment of planets as viewed from Earth. Back in May of 2002 the planets Mercury, Venus, Mars, Saturn and Jupiter formed a near line across the sky from the horizon to near zenith (directly overhead). This is a once or twice in a lifetime event—not occurring again until 2040.

Another use of the term "planetary alignment" is defined as the planets being lined up in a row, outward from the sun, as viewed from the sun. This is what most people associate with the term "planetary alignment". Actually, such an even cannot happen as the orbital planes of the planets are not perfectly coplanar—they are all tilted at different angles. With this kind of "planetary alignment", we are satisfied if the planets lie in the same quadrant of the sky—all within 90° of each other. In Part 1 of this classroom application, you will investigate this kind of an alignment with three Ozobot Bits programmed to behave just like real planets on the provided OzoMap.

In Part 2 of this classroom application, you will investigate Kepler's Law of Periods with your three programmed Ozobot Bits and OzoMap. This law states that *the ratio of the square of the orbital period of a planet to the cube of its average distance from the sun is the same for all of the planets*. Johannes Kepler was able to find this out empirically in the early 1600's by careful examination of data from his mentor, Tycho Brahe. This Law of periods was shown in the later 1600's to be a consequence of Sir Isaac Newton's laws of motion and his law of universal gravitation.

The OzoMap and OzoBlockly Programs for this Classroom Application

The last page of this document contains the OzoMap needed for this classroom application. It shows a star at the center and the orbits of three planets (Planet Alpha, Planet Beta, and Planet Gamma) around this star. The orbits are circular and coplanar, with all orbits lying on the surface of the paper. The orbits are divided into twelve "pie" slices, each 30° in size. These slices will be used by students to determine when the three planets are in reasonably close alignment, all within 30° or 60°, with 30° the "closest" alignment category. The three orbits are labeled red, green, and blue. These colors correspond to the colors displayed by the OzoBlockly programs for the three Ozobot Bits that represent planets Alpha, Beta, and Gamma:

- *PlanetAlpha.ozocode* (Red)
- *PlanetBeta.ozocode* (Green)
- *PlanetGamma.ozocode* (Blue)

After loading the three OzoBlockly programs onto the three Ozobot Bits, ***it is important that the Ozobot Bits be placed on the correct orbits***, based upon the colors displayed and the color name labels on the OzoMap. (The OzoBlockly programs assign speeds that are in agreement with the speeds required for the radii of the orbits, all in agreement with Kepler's Law of Periods.)

You will note that the supplied OzoMap has orbits that are half the width required for Ozobot line following. After printing this OzoMap on an 8½" x 11" sheet of paper, you will need to take it to an office products store (such as OfficeMax) and have them make a 200% gray-scale enlargement on paper that is 17" x 22". There is no need to go with an expensive color enlargement, but a ***gray-scale*** enlargement provides better resolution than straight black-and-white. The enlarged OzoMap will have lines that are just the right width for Ozobot line following. (The enlargement is necessary because the orbits are too close together on the 8½" X 11" sheet. The Ozobot Bits are so close together that they would bump one another when trying to pass each other in their orbits.)

Preparation for this Classroom Application

Either Part 1 (Planetary Alignment), Part 2 (Kepler's Law of Periods), or both can be used in this classroom application.

1. Prepare a 17" x 22" OzoMap as indicated in the previous section of this document for each lab group. Remember that each lab group will need three Ozobot Bits, so you may need to combine smaller lab groups into a larger lab group.
2. If Part 1 of this classroom application is on your agenda, then you will need to make a copy of pages 3 through 5 for each lab group. Each lab group will also need a stop watch or stop watch app on their cell phone.
3. If Part 2 of this classroom application is on your agenda, then you will need to make a copy of page 3 and page 7 for each lab group.
4. All Ozobot Bits should be fully charged, calibrated, and have clean wheels.
5. If Part 2 is on your agenda, each lab group will need:
    a. A centimeter ruler to measure orbital radii.
    b. A stop watch to measure Ozobot Bit orbital times, preferably to hundredths of a second. Alternatively, students could use a stop watch app on their cell phones.
6. Either have the students load the three OzoBlockly programs as described in the previous section of this document, or load them in advance to save class time. (The programs are very short, so each one takes less than 30 seconds to load.)
7. To start their Ozobot Bits, students should double-press the start button. If the programs are loaded correctly, one of the Bits should display red, another green, and the remaining Bit blue.

With all of the preparation accomplished, your students can begin an interesting and exciting investigation of planetary and space science concepts!

Setting the Stage: The Time, Place, and Situation

It is Earth year 2515 in a distant galaxy in the Milky Way. The *Intergalactic Survey Team* (IST) has recently discovered a small planetary system surrounding a star not much unlike our Sun. A study of this planetary system has revealed a number of interesting facts. It has three planets with orbits that are amazingly coplanar, significantly more coplanar than those of the nine planets orbiting the Sun. (Due to newly discovered information and popular demand, Pluto was reclassified as a planet back in the year 2073.) Not unlike Earth's solar system, all three planets orbit in the same direction around their star. The orbits are also very nearly circular, with orbital eccentricities less than that of Earth's 0.02. The orbital radii of these three planets have been carefully determined by the IST. Finally, the IST has found that these three planets line up with regularity on the same side of their star to within 30° to 60° of each other.

The star has been named OzoSol, following the current trend of naming newly discovered stars after well-known Earth toys that have survived the test of time. Its three planets have been named Alpha (α), Beta (β), and Gamma (γ), in order of their distance from the star, after the first three letters of the Greek alphabet. An Ozobot Bit based scale model of this planetary system has been constructed—with each millimeter representing a million miles and Ozobot Bits acting as planets Alpha, Beta , and Gamma. A small diagram of this model is shown in Figure 1.

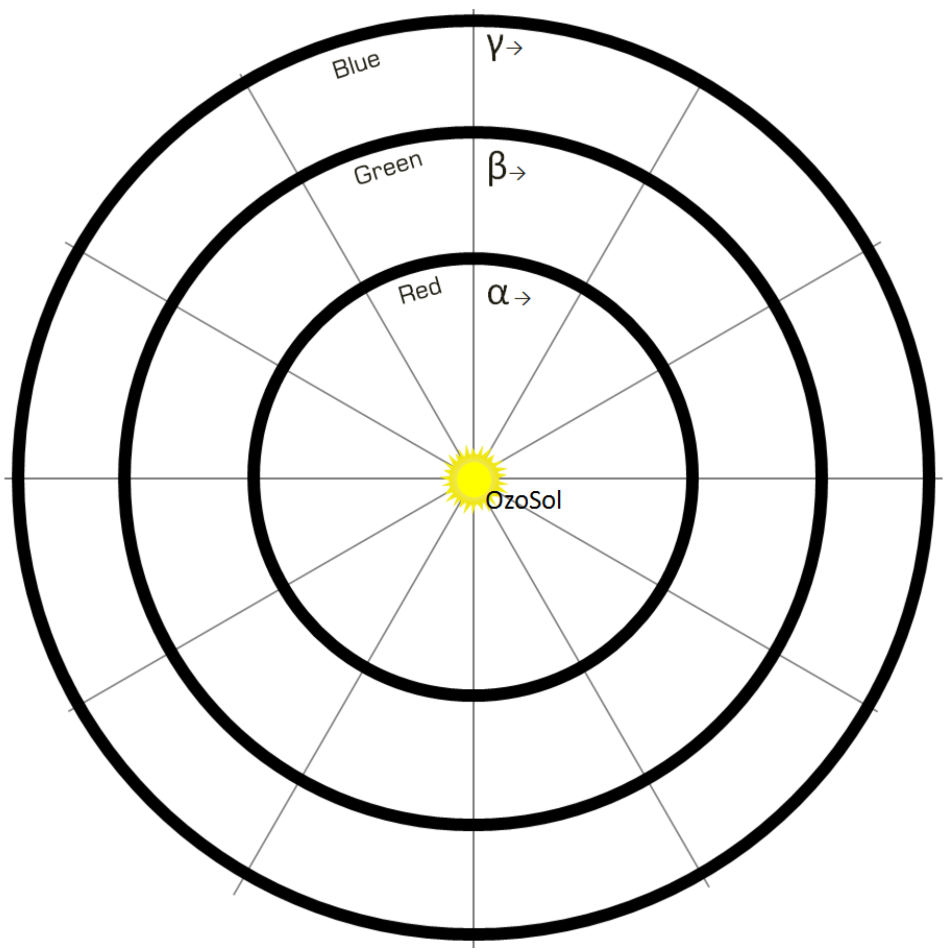


Figure 1

Part 1: Planetary Alignment

After loading the three Ozobot Bit programs (*PlanetAlpha.ozocode, PlanetBeta.ozocode, and PlanetGamma.ozocode*) onto three separate Ozobot Bits, start them by double-pressing their start buttons. You should notice that one shows a red LED, another shows a green LED, and the third shows a blue LED. Place them on their respective orbits (labeled red, green, and blue) on the large OzoMap so that they are all moving clockwise, as indicated by the three arrows on the OzoMap. Hold a block of wood or a book along the vertical gray line at the top of the OzoMap. When all three planets (represented by the Ozobot Bits) are pushing against the wood block or book, quickly remove it so the planets are free to move, and ***simultaneously start a stopwatch***. This way the planets are lined up perfectly at time zero.

Your objective is to determine the average time between successive "alignments" of all three planets on the same side of OzoSol. The orbital plane is divided into twelve slices of 30° each. You will be recording the stop watch time each time there is an "alignment" and will record the alignment as either 30° or 60°, depending upon how close the alignment is. Alignment should be based upon location of the *centers* of the Ozobot planets. In other words, are the *centers* of the planets within 30° or 60°? Record alignment data on the provided data table for 12 to 15 minutes. Compute the average time between successive alignments and record this value at the bottom of the table. When all groups have completed filling in the data table, your teacher will probably initiate a discussion by comparing the group averages on a white board.

Discussion Questions

1. Were the times between "alignments" reasonably regular, or was there high variation in these times?
2. What was the shortest time between "alignments"? What was the longest time?
3. If each second on the OzoSol model corresponds to 30 Earth days, what is the average number of days between mutual "alignment" of Planets Alpha, Beta, and Gamma?
4. Were your alignments close to being equally distributed between the 30° and 60° categories?
5. Did you find it fairly easy to identify 30° and 60° alignments, or were there things that made this a bit challenging?
6. Were the average times from different lab groups similar? What are some possible reasons for differences?

Part 1: Planetary “Alignment” Data Table

Stop Watch Time		Time Between Successive Alignments (sec)	“Alignment” Category 30° or 60°
mm:ss	seconds		
0:00	0	------	Perfect Alignment
	Average→		

For the Teacher: Typical Results

Part 1: Planetary "Alignment" Data Table

Stop Watch Time		Time Between Successive Alignments (sec)	"Alignment" Category 30° or 60°
mm:ss	seconds		
0:00	0	------	Perfect Alignment
0:59	59	59	30
1:58	118	59	60
2:39	159	41	60
3:37	217	58	30
4:30	270	53	60
5:19	319	49	30
6:17	377	58	30
7:14	434	57	60
7:56	476	42	60
8:52	532	56	30
9:48	588	56	60
10:34	634	46	60
11:29	689	55	30
12:28	748	59	60
13:10	790	42	30
14:09	849	59	30
14:53	893	44	60
	Average→	53	

Part 2: Kepler's Law of Periods

After loading the three Ozobot Bit programs (*PlanetAlpha.ozocode, PlanetBeta.ozocode, and PlanetGamma.ozocode*) onto three separate Ozobot Bits, start them by double-pressing their start buttons. You should notice that one shows a red LED, another shows a green LED, and the third shows a blue LED. Place them anywhere on their respective orbits (labeled red, green, and blue) of the large OzoMap so that they are all moving clockwise, as indicated by the three arrows on the OzoMap.

Your objective is to verify Kepler's Law of Periods, which states that *the ratio of the square of the orbital period of a planet to the cube of its average distance from the sun is the same for all of the planets.* Therefore, you will need to use a metric ruler to measure the radii of the three planetary orbits to the nearest tenth of a cm. To determine the ***period***, *i.e.,* the time for one revolution of a planet about its star, you will measure the time for five orbits and then divide this time by 5. All of your data can be recorded in the provided data table below.

Part 2: Kepler's Law of Periods Data Table

Planet	Time for 5 Orbits (sec)	Orbital Period, T (sec)	Orbital Radius, R (cm)	T^2 (sec^2)	R^3 (cm^3)	The Ratio T^2/R^3 (sec^2/cm^3)
Alpha (α)						
Beta (β)						
Gamma (γ)						
					Average→	

Questions

1. Does your data seem to verify Kepler's Law of Periods for the OzoSol planetary system? Why or why not?
2. Using your value of the average ratio for T^2/R^3, assuming that 1 mm on the OzoMap corresponds to one million miles, and assuming that 1 second on your stop watch corresponds to 30 Earth days:
    a. What would be the period of a planet around OzoSol if it was orbiting at a radius of 300 million miles?
    b. What would be the orbital radius of a planet orbiting OzoSol with an orbital period of 163 days?

For the Teacher: Typical Results

Part 2: Kepler's Law of Periods Data Table

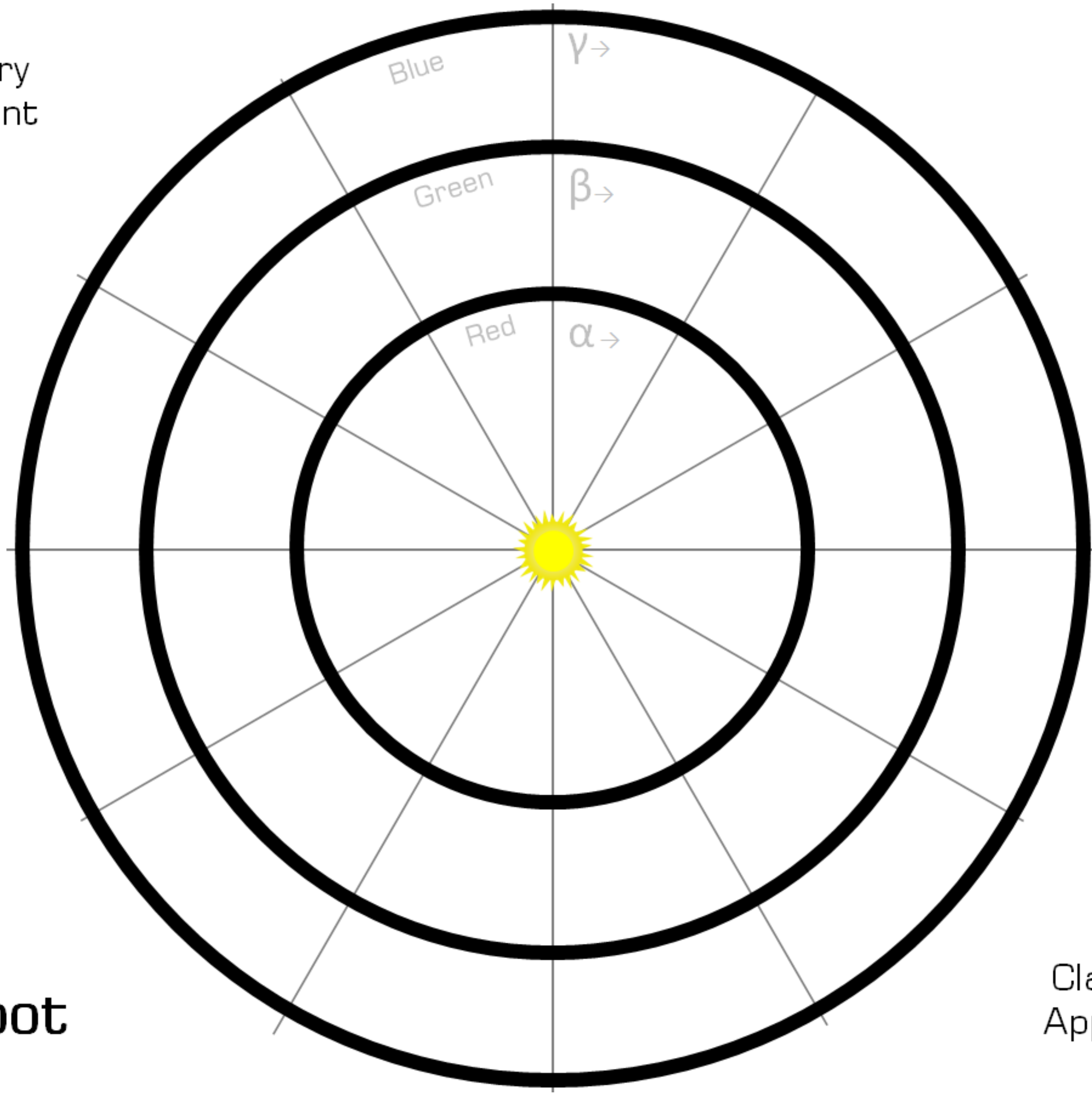
Planet	Time for 5 Orbits (sec)	Orbital Period, T (sec)	Orbital Radius, R (cm)	T^2 (sec^2)	R^3 (cm^3)	The Ratio T^2/R^3 (sec^2/cm^3)
Alpha (α)	48.55	9.71	8.9	94.28	704.97	0.13
Beta (β)	90.31	18.1	14.2	327.61	2863.29	0.11
Gamma (γ)	134.14	26.83	18.6	719.84	6434.86	0.11
					Average→	0.117

Answers to Questions (based upon data from the above table):

1. Results should be consistent with Kepler's Law, with the ratio of T^2/R^3 reasonably constant.
2. Calculations would work out as follows:
   (a) 300 Million miles corresponds to 300 mm = 30 cm on the OzoMap. $T^2/30^3 = 0.117$. Solving this equation for T, we find that T = 56.2 sec on the OzoMap model. 56.2 sec x 30 days/sec = 1686 days.
   (b) 163 days corresponds to 163/30 = 5.43 sec on the OzoMap. $5.43^2/R^3 = 0.117$. Solving this equation for R, we find that R = 6.32 cm on the OzoMap. Since 6.32 cm = 63.2 mm, then the orbital radius would be 63.2 Million miles.

Planetary Alignment

Kepler's Law of Periods



Ozobot

Classroom Application